



State University of Ponta Grossa, 4th of September 2023
Centro Integrar, Room 9

TIMETABLE

Time	Title	Speaker
Seminar		
10:00 - 11:15	Point Potentials in One Dimension	Manuel Gadella <i>UVa, Spain</i>
Oral Communications		
11:20 - 11:55	Analytically Solvable Time-Dependent Hamiltonians with SU(2) and SU(1,1) Symmetries	Antônio S M de Castro <i>DEFIS - UEPG</i>
12:00 - 13:30	LUNCH	
13:35 - 14:10	Physical Interpretation of the Parameters in One Dimensional Relativistic Point Interactions	José Tadeu Lunardi <i>DEMAT - UEPG</i>
14:15 - 14:50	The Dirac equation in singular multibarriered arrangements	Carlos A Bonin <i>Independent Scholar</i>
14:55 - 15:30	Scattering entropies of quantum graphs	Fabiano M de Andrade <i>DEMAT - UEPG</i>
15:30 - 16:00	COFFE BREAK	
16:05 - 16:40	One-Dimensional Quantum Scattering via Green's Function and Continued Fractions	Luna R N Oliveira <i>PhD Student - UFPR</i>
16:45 - 17:20	SELF ADJOINT EXTENSIONS OF THE MAGNETIC SCHRÖDINGER OPERATOR AND GAUGE EQUIVALENCE IN NON SMOOTH DOMAINS	Wagner Monteiro <i>DEMAT-UEPG</i>

ABSTRACTS

- Manuel Gadella (Uva, Spain) – **Point Potentials in One Dimension**
 - *To be announced*
- A. S. M. de Castro, R. Grimaudo, D. Valenti, A. Migliore, H. Nakazato and A. Messina- **Analitically Solvable Time-Dependent Hamiltonians with SU(2) and SU(1,1) Symmetries**
 - *Since nonstationary quantum systems are described by time-dependent Hamiltonian operators, solving the Liouville-Cauchy problem that corresponds to the evolution operator is generally not an easy task. As a consequence, in many physical scenarios a perturbative or numerical approach is required to describe the time evolution of the quantum state of a system. In the case of finite dimension quantum systems described by time-dependent Hamiltonians with SU(2) dynamical symmetry, the elegant scheme proposed in Ref. [1] (MN ansatz) allows one to determine different classes of analytically solvable SU(2) Hamiltonians. The semiclassical Rabi model [2], which involves a two-level quantum system, was exactly solved, and two different parameterizations were obtained for the solutions of the correspondent evolution operator [3]. The same scheme was also considered for a time-dependent SU(1, 1) Hamiltonian operator with PT-symmetry. The physical relevance to the context of the gain-loss mechanism was considered in describing the dynamics of two coupled waveguides [4]. The method can also be applied to the case of a finite-dimension coupled quantum system, where internal symmetry properties allow reducing the physical problem to the study of independent fictitious systems exactly solvable in their dynamically invariant Hilbert spaces [5]. In the context of an infinite-dimension quantum system (the general case of the time-dependent Jayne-Cummings model for a two-level atom), we discuss the applicability of the MN ansatz to find the exact solution for the evolution operator of the full system. The Hilbert space is split into a direct sum of infinitely many dynamically invariant subspaces. Then, in the resonance regime, it is possible to determine the overall dynamics of the system by applying the MN ansatz to a single subspace.*

References

[1] A. Messina and H. Nakazato, *J. Phys. A: Math. Theor.* 47, 445302 (2014).
[2] I. I. Rabi, *Phys. Rev.* 49, 324 (1936).
[3] R. Grimaudo, A. S. M. de Castro, H. Nakazato, and A. Messina, *Ann. Phys.* 530, 1800198 (2018).
[4] R. Grimaudo, A. S. M. de Castro, H. Nakazato, and A. Messina *Phys. Rev. A*, 052103 (2019).
- José Tadeu Lunardi (DEMAT - UEPG), Carlos A Bonin (Independent Scholar), Luiz A. Manzoni (Concordia College) - **Physical Interpretation of the Parameters in One Dimensional Relativistic Point Interactions**
 - *We investigate point interactions in one-dimensional relativistic quantum mechanics using a distributional approach based on Schwartz's theory of distributions. From the properties of the most general covariant distribution describing relativistic point interactions we obtain the physical parameters*

associated with the point potentials that behave as a scalar, a pseudo-scalar and a vector under Lorentz transformations. Then, we establish a one-to-one relationship between these physical parameters and the parameters of the self-adjoint extensions of the Hamiltonian. By considering the non-relativistic limit, we obtain the most general point interaction in the Schrödinger equation in terms of these four physical point potentials. Finally, we study the symmetries of the relativistic point interactions under space inversion, time reversal and charge conjugation, and investigate how requirements of invariance under these symmetry transformations can be used to restrict the set of physical parameters

- Carlos A Bonin (Independent Scholar), José T. Lunardi (UEPG), Luiz A. Manzoni (Concordia College) – **The Dirac equation in singular multibarriered arrangements**
 - In this work, we present a mathematically rigorous investigation of the Dirac equation interacting with one-dimensional spatial arrangements of singular point interactions within Schwartz's distribution theory's framework. Our focus is the study of two point barriers in both odd and even configurations, taking into account the physical interpretation of these barriers in terms of scalar, pseudo-scalar, and vector singular Lorentz fields. We also investigate the phenomena of resonances in two-barrier configurations - conditions, these, under which the transmission coefficient equals 1. Through a limiting process, we explore the theoretical collapse of these two barriers into a singular one. Furthermore, preliminary results for an arrangement of N point interactions at equidistant intervals are discussed.
- Fabiano M de Andrade (DEMAT - UEPG) – **Scattering entropies of quantum graphs**
 - *The scattering amplitude in simple quantum graphs is a well-known process which may be highly complex. In this presentation, motivated by the Shannon entropy, we propose a methodology that associates to a graph a scattering entropy, which we call the average scattering entropy. It is defined by taking into account the period of the scattering coefficient which we calculate using the Green's function approach. We first describe the methodology on general grounds, and then exemplify our findings considering several distinct types of graphs. In the end, we extend our proposal to the case of Rényi and Tsallis entropies*
- Luna R. N. Oliveira (UFPR), Marcos G. E. da Luz (UFPR) - **One-Dimensional Quantum Scattering via Green's Function and Continued Fractions**
 - *An essential problem in physics is the nonrelativistic quantum scattering in one-dimensional networks, not just due to the vast number of applications — like those in condensed matter and quantum computation — but due to the several mathematical methods that emerged intending to solve the issue. In this work, we propose an analytic method that uses the exact Green function to obtain recurrence relations for the scattering amplitudes. The recurrence relations are constructed in such a way that is possible to employ continued fractions to determine the transmission and reflection coefficients for any barrier formed by a sequence of N building blocks, just by knowing the scattering amplitudes of a single building block. A great advantage of this approach is that it can be applied to a large number of periodic potentials. Considering the substantial number of applications, we employ three exact and analytically solved*

potentials as a basis: Dirac delta, square wall, and trapezoidal wall, and we study the scattering effects of the possible combinations between them, especially the behavior of the transmission probability for different potential parameters and incident wavenumbers.

- Wagner Monteiro (UEPG) - **SELF ADJOINT EXTENSIONS OF THE MAGNETIC SCHRÖDINGER OPERATOR AND GAUGE EQUIVALENCE IN NON SMOOTH DOMAINS**

- Using boundary triples, we obtained a parametrization for all self-adjoint realization of the magnetic Schrödinger operator, in a quasi-convex domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$ with compact boundary, and magnetic potentials with components in $W_\infty^1(\overline{\Omega})$. Then we apply this parametrization and generalize a characterization of the gauge equivalence of the Dirichlet magnetic operator with the Dirichlet Laplacian, obtaining a condition for the gauge equivalence of a general self-adjoint realization of the magnetic Schrödinger operator and a self-adjoint realization of the free kinetic energy operator .

Keywords: magnetic Laplacian; self-adjoint extensions; quasi-convex domains; gauge transformations.

Bibliografia

[1] Grubb, G., (1968) A Characterization of non-local boundary value problems associated with an elliptic operator. *Ann. Scuola Norm. Sup. Pisa* (3) 22, 425–513.

[2] Gesztesy, F., Mitrea, M., (2011) A Description of all self-adjoint extensions of the Laplacian and Krein-Type resolvent formulas on non-smooth domains. *J. Anal. Math.* 113, 53–172.

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[4] de Oliveira, C. R., Monteiro, W., (2021) All self-adjoint extensions of the magnetic Laplacian in non smooth domains and gauge transformations. *Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) Vol. XXII* (2021), 1805-1841.